Breaking of Energy Conservation Law: Creating and Destroying of Energy by Subwavelength Nanosystems

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The *extra energy*, *negative energy* and *annihilation of energy* by the subwavelength conservative systems that have a wave nature of light or matter (quantum) objects are predicted. The creating and destroying of energy break the energy conservation law in any subwavelength physical system. The paradoxical phenomenon is demonstrated in the context of extraordinary transmission of light and matter through subwavelength apertures [T.W. Ebbesen et al., Nature (London) 391, 667 (1998) and E. Moreno et al., Phys. Rev. Lett. 95, 170406 (2005)].

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The energy conservation law is the most important of conservation laws in physics. Conservation of energy states that the total amount of energy in a isolated system remains constant. In other words, energy can be converted from one form to another, but it cannot be created or destroyed. The energy conservation law is a mathematical consequence of the shift symmetry of time; energy conservation is implied by the empirical fact that physical laws remain the same over time. The energy conservation affects all physical phenomena without exceptions, for an example, the recently discovered extraordinary (enhanced) transmission of light through subwavelength apertures in a metal screen [1, 2, 3, 4, 5, 6, 7, 8]. The transmission enhancement is a process that can include the resonant excitation of surface plasmons [2, 3, 4], Fabry-Perot-like intraslit modes [5, 6, 7], and evanescent electromagnetic waves at the metal surface [8]. In the case of thin screens whose thickness are too small to support the intraslit resonance, the extraordinary transmission of light or matter (electrons) is caused by the resonant excitation of surface waves [2, 3, 4, 9]. At the resonant conditions, the system redistributes the electromagnetic energy around the screen, such that more energy is effectively transmitted compared to the energy impinging on the slit opening. The total energy of the system is conserved under the energy redistribution. In the present paper, we predict the *extra energy*, *negative energy* and *annihilation of energy* by an ensemble of light or matter beams produced by an array of subwavelength apertures. The creating and destroying of energy break the energy conservation law in any subwavelength physical system. The phenomenon, in particular, is associated with the extraordinary transmission without assistance of the surface waves.

Let us first investigate the transmission of light through a subwavelength structure, namely an array of parallel subwavelength slits. The array of M independent slits of width $2a$ and period Λ in a metallic screen of thickness $b \ll \lambda$ is considered. The metal is assumed to be a perfect conductor. The screen placed in vacuum is illuminated by a normally incident TM-polarized wave with wavelength $\lambda = 2\pi c/\omega = 2\pi/k$. The magnetic field of the wave $\vec{H}(x, y, z, t) = U(x)\exp(-i(kz + \omega t))\vec{e}_y$ is assumed to be time harmonic and constant in the y direction. The energy balance, which determines the transmission coefficient of the slit array, is derived by calculating the power of light beams in the far-field diffraction zone. The EM beams produced by each of the independent slits are computed by using the Neerhoff and Mur approach, which uses a Green's function formalism for rigorous numerical solution of Maxwell's equations for a single, isolated slit [10, 11]. The transmission of the slit array is determined by calculating all the light power $P(\lambda)$ radiated into the far-field diffraction zone, $x \in [-\infty, \infty]$ at the distance $z \gg \lambda$ from the screen. The total per-slit transmission coefficient, which represents the per-slit enhancement in transmission achieved by taking a single, isolated slit and placing it in an M-slit array, is then found by using an equation $T_M(\lambda) = P(\lambda)/MP_1$. Here, P is the total power of M beams produced by the array, and P_1 is the power of a beam produced by the single slit. Figure 1 shows the transmission coefficient $T_M(\lambda)$, in the spectral region 500-2000 nm, calculated for the array parameters: $a = 100$ nm, $\Lambda = 1800$ nm, and $b = 5 \times 10^{-3}\lambda_{max}$. The transmitted power was computed by integrating the total energy flux at the distance $z = 1$ mm over the detector region of width $\Delta x = 20$ mm. The transmission spectra $T_M(\lambda)$ is shown for different values of M. We notice that the spectra $T_M(\lambda)$ is periodically modulated, as a function of wavelength, below and above a level defined by the transmission $T_1(\lambda) = 1$ of one isolated slit. As M is increased from 2 to 10, the visibility of the modulation fringes increases approximately from 0.2 to 0.7. The transmission T_M exhibits the Fabry-Perot like maxima around wavelengths $\lambda_n = \Lambda/n$ (n=1, 2, ...). The spectral peaks increase with increasing the number of slits and reach a saturation ($T_M^{max} \approx 5$) in amplitude by $M = 300$, at $\lambda \approx 1800$ nm. The peak widths and the spectral shifts of the resonances from the Fabry-Perot wavelengths decrease with increasing the number M of slits. Figure 1 indicates that enhancement and suppression in the transmission spectra are the natural proper-

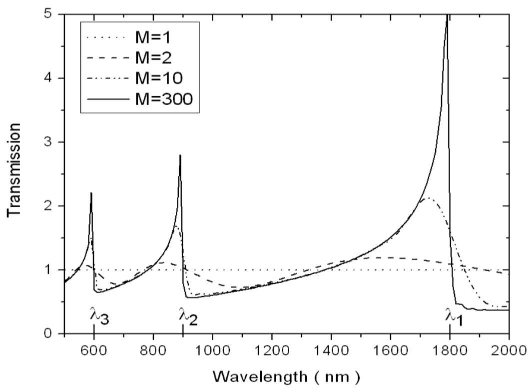

FIG. 1: The per-slit transmission $T_M(\lambda)$ of an array of independent slits of the period Λ versus the wavelength for different number M of slits. There are three Fabry-Perot like resonances at the wavelengths $\lambda_n \approx \Lambda/n$, n=1, 2 and 3.

ties of an ensemble of independent subwavelength slits in a thin ($b \ll \lambda$) screen. The spectral peaks are characterized by asymmetric Fano-like profiles. Such modulations in the transmission spectra are known as Wood's anomalies. The minima and maxima correspond to Rayleigh anomalies and Fano resonances, respectively. The Wood anomalies in transmission spectra of optical gratings, a long standing problem in optics [12], follows naturally from interference properties of our model. The new point is a weak Wood's anomaly in a classical Young type two-slit system ($M = 2$). Figure 1 shows the *extra energy* ($T > 1$), *negative energy* ($T < 1$) and *annihilation of energy* ($T < 1$). The creating and destroying of energy break the energy conservation law in the system of M independent subwavelength beams (slits).

To clarify the results of the computer code we have developed an analytical model, which yields simple formulas for the diffracted fields. For the fields diffracted by a narrow ($2a \ll \lambda, b \geq 0$) slit into the region $|z| > 2a$, it can be shown that the Neerhoff and Mur model simplifies to an analytical one. For the magnetic $\vec{H} = (0, H_y, 0)$ and electric $\vec{E} = (E_x, 0, E_z)$ fields we found:

$$H_y(x,z) = iaDF_0^1(k[x^2+z^2]^{1/2}), \tag{1}$$

$$E_x(x,z) = -az[x^2+z^2]^{-1/2}DF_1^1(k[x^2+z^2]^{1/2}), \tag{2}$$

and

$$E_z(x,z) = ax[x^2+z^2]^{-1/2}DF_1^1(k[x^2+z^2]^{1/2}), \tag{3}$$

where

$$D = 4k^{-1}[[\exp(ikb)(aA-k)]^2 - (aA+k)^2]^{-1} \tag{4}$$

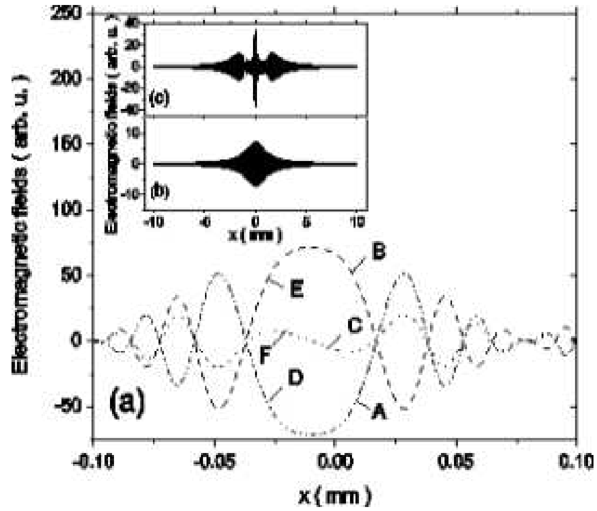

FIG. 2: Electromagnetic fields in the far-field zone. (a) The fields $\mathrm{Re}(E_x(x))$ (A and D), $\mathrm{Re}(H_y(x))$ (B and E), and $\mathrm{Re}(10E_z(x))$ (C and F) calculated for $M = 10$ and $\lambda = 1600$ nm. The curves A, B, and C: rigorous model; curves D, E, and F: analytical model. (b) $\mathrm{Re}(E_x(x))$ for M=1: analytical model. (c) $\mathrm{Re}(E_x(x))$ for M=5: analytical model.

and

$$A = F_0^1(ka) + \frac{\pi}{2}[\bar{F}_0(ka)F_1^1(ka) + \bar{F}_1(ka)F_0^1(ka)]. \tag{5}$$

Here, F_1^1, F_0^1, $\bar{F}_0$ and $\bar{F}_1$ are the Hankel and Struve functions, respectively. The fields are spatially nonuniform, in contrast to a common opinion that a subwavelength aperture diffracts light in all directions uniformly [13]. The fields produced by an array of M independent slits are given by $\vec{E}(x,z) = \sum_{m=1}^{M} \vec{E}_m(x+m\Lambda, z)$ and $\vec{H}(x,z) = \sum_{m=1}^{M} \vec{H}_m(x+m\Lambda, z)$, where $\vec{E}_m$ and $\vec{H}_m$ are the fields of an m-th beam generated by the respective slit. As an example, Fig. 2(a) compares the far-field distributions calculated by the analytical formulas (1-5) to that obtained in the rigorous model. We notice that the distributions are indistinguishable. The field power P, which determines the field energy, is found by integrating the energy flux $|\langle\vec{S}\rangle_t| = |\vec{E}\times\vec{H}^* + \vec{E}^*\times\vec{H}|(c/16\pi)$. Thus, the analytical model describes accurately also the energy balance in the system of M independent subwavelength beams. The model does not only support results of our computer code, but presents an intuitively transparent explanation (physical mechanism) of the *extra energy*, *negative energy*, and *annihilation of energy* in terms of the constructive or destructive interference of the M independent subwavelength beams produced by the multi-beam source. The creating and destroying of energy, which are associated with the extraordinary transmission without assistance of the surface waves, break the energy conservation law. Notice that the array-induced decrease of the central beam divergence (Figs. 2(b) and 2(c)) is

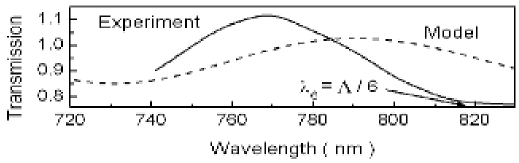

FIG. 3: The per-slit transmission coefficient $T(\lambda)$ versus wavelength for the Young type two-slit experiment [16]. Solid curve: experiment; dashed curve: analytical model. Parameters: $a = 100$ nm, $\Lambda = 4900$ nm, and $b = 210$ nm.

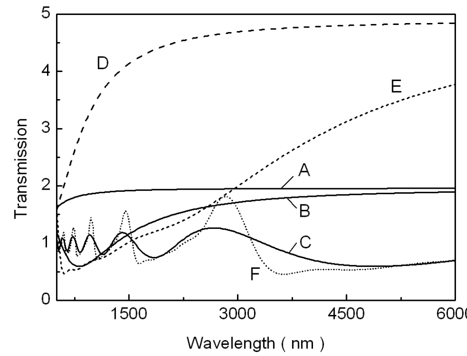

FIG. 4: The per-slit transmission $T_M(\lambda)$ versus wavelength for the different values of Λ and M: (A) $\Lambda = 100$ nm, $M = 2$; (B) $\Lambda = 500$ nm, $M = 2$; (C) $\Lambda = 3000$ nm, $M = 2$; (D) $\Lambda = 100$ nm, $M = 5$; (E) $\Lambda = 500$ nm, $M = 5$; (F) $\Lambda = 3000$ nm, $M = 5$. Parameters: $a = 100$ nm and $b = 10$ nm. There are two enhancement regimes at $\Lambda \ll \lambda$ and $\Lambda \geq \lambda$.

relevant to the beaming light [14], and the diffraction-free light and matter beams [15]. The amplitudes of the beams (evanescent fields) can rapidly decrease with increasing the distances x and z. However, due to the enhancement and beaming mechanisms (Figs. 1-4), the array produces a propagating wave with low divergence. Such a behavior is in agreement with the Huygens-Fresnel principle, which considers a propagating wave as a superposition of secondary spherical waves.

It is now important to understand the energy balance in the two fundamental systems of wave optics, the single-slit and two-slit systems. The major features of the transmission through a single subwavelength slit are the intraslit resonances and the spectral shifts of the resonances from the Fabry-Perot wavelengths [5]. In agreement with the predictions [5], the formula (4) shows that the transmission $T = P/P_0 = (a/k)[\mathrm{Re}(D)]^2 + [\mathrm{Im}(D)]^2$ exhibits Fabry-Perot like maxima around wavelengths $\lambda_n = 2b/n$, where P_0 is the power impinging on the slit opening. The enhancement and spectral shifts are explained by the wavelength dependent terms in the denominator of Eq. (4). The enhancement $(T(\lambda_1) \approx b/\pi a$ [15]) is in contrast to the attenuation predicted by the model [5]. At the resonant conditions, the system redistributes the electromagnetic energy in the intra-slit region and around the screen, such that more energy $(T > 1)$ is effectively transmitted compared to the energy impinging on the slit opening. The total energy of the system is conserved under the energy redistribution. This mechanism is different from those based on the creating and destroying of energy by the multi-beam (multi-slit) system. The Young type two-slit configuration is characterized by a sinusoidal modulation of the transmission spectra $T_2(\lambda)$ [16, 17]. The modulation period is inversely proportional to the slit separation Λ. The visibility V of the fringes is of order 0.2, independently on the slit separation. In our model, the transmission is given by $T_2 \sim \int [F_1^1(x_1)[iF_0^1(x_2)]^* + F_1^1(x_1)^* iF_0^1(x_2)]dx$, where $x_1 = x$ and $x_2 = x + \Lambda$. The high-frequency modulations with the sideband-frequency $f_s(\Lambda) \approx f_1(\lambda) + f_2(\Lambda, \lambda) \sim 1/\Lambda$ (Figs. 1 and 3) are produced like that in a classic heterodyne system by mixing two waves having different spatial frequencies, f_1 and f_2. Although our model ignores the plasmons, its prediction for the transmission ($T_2^{max} \approx 1.1$), the visibility ($V \approx 0.1$) of the fringes and the resonant wavelengths $\lambda_n \approx \Lambda/n$ compare well with the plasmon-assisted Young's type experiment [16] (Fig. 3). In the case of $b \geq \lambda/2$, the resonances at $\lambda_n = \Lambda/n$ can be accompanied by the intraslit resonances at $\lambda_n = 2b/n$. One can easily demonstrate such behavior by using the analytical formulas (1-5). We considered the TM polarization because TE modes are cut off by a thick slit. In the case of a thin screen, TE modes propagate into slits so that magneto-polaritons develop. Because of the symmetry of Maxwell's equations the scattering intensity is formally identical with $\vec{E}$ and $\vec{H}$ swapping roles. The described mechanism is not the only contribution to enhanced transmission. There can be also enhancement due to the energy redistribution by surface waves [2, 3, 4, 8, 16, 17, 18]. The surface waves can couple the radiation phases of the slits, so that they get synchronized, and a collective emission can release the stored energy as an enhanced radiation. This kind of enhancement is of different nature compared to our model, because the model does not require coupling between the beams.

In order to gain physical insight into the energy balance in the multi-wave ($M \geq 2$) systems, we now consider the dependence of the transmission $T_M(\lambda)$ on the slit separation Λ. We assume that the slits (beams) are independent also at $\Lambda \to 0$. According to the Van Citter-Zernike coherence theorem, a light source (even incoherent) of radius $r = M(a + \Lambda)$ produces a transversally coherent wave at the distance $z \geq \pi R r/\lambda$ in the region of radius R. Thus, in the case of a subwavelength light source ($\Lambda \ll \lambda$), the collective emission of the ensemble of slits generates the coherent electric and mag-

netic fields, $\vec{E} = \sum_{m=1}^{M} \vec{E}_m \exp(i\varphi_m) \approx M\vec{E}_1 \exp(i\varphi)$ and $\vec{H} \approx M\vec{H}_1 \exp(i\varphi)$. This means that the beams arrive at the detector with the same phases $\varphi_m(x) \approx \varphi(x)$ (see, also Ref. [19]). Consequently, the power (energy) of the emitted light scales with the square of the number of slits (beams), $P \approx M^2 P_1$. Therefore, the transmission ($T_M = P/MP_1$) grows linearly with the number of slits, $T_M \approx M$. For a given M, the function $T_M(\lambda)$ monotonically varies with λ (Fig. 4). At the appropriate conditions, the transmission can reach the 1000-times enhancement ($M = \lambda z/\pi R(a+\Lambda)$). In the case of $R \geq \lambda z/\pi r$ ($\Lambda \geq \lambda$), the beams arrive at the detector with different phases $\varphi_m(x)$. Consequently, the power and transmission grow more slow with the number of beams (Figs. 1-4). Notice, that according to the energy conservation law one should expect that the energy (transmission) would remain constant with changing the slit (beam) separation. The creating and destroying of energy in a wave field, which are associated with the extraordinary transmission, break the conservation law. In the case of $\Lambda >> \lambda$, our model is in agreement with the conservation law and theories of gratings [20]. Our consideration of the subwavelength gratings is similar in spirit to the dynamical diffraction models [21, 22], the Airy-like model [23], and especially to a surface evanescent wave model [8].

One can easily demonstrate the breaking of energy conservation in any subwavelength physical system by taking into account the interference properties of Young's double-source system. At the risk of belaboring the obvious, we now describe the phenomenon. In the far-field diffraction zone, the radiation from two pinholes of Young's setup is described by two spherical waves. The light intensity at the detector is $I(\vec{r}) = |(E/r_1)exp(ikr_1+\varphi_1)+(E/r_2)exp(ikr_2+\varphi_2)|^2 = I_1+I_2+2(I_1I_2)^{1/2}cos([kr_1+\varphi_1]-[kr_2+\varphi_2])$. The corresponding energy is $W = \int\int(I_1 + I_2 + 2(I_1I_2)^{1/2}cos([kr_1+\varphi_1] - [kr_2+\varphi_2])dxdy$. Here, we use the units $c\Delta t/8\pi = 1$. In conventional Young's setup, which contains the pinholes separated by the distance $\Lambda >> \lambda$, the interference cross term (energy) vanishes. In accordance with the conservation law, the energy is $W = \int\int(I_1+I_2)dxdy = W_1 + W_2 = 2W_0$, where $W_1 = W_2 = W_0$. In the case of Young's subwavelength system ($\Lambda << \lambda$; $r_1 = r_2$ for any coordinate x or y), the energy is $W = W_1 + W_2 + 2\int\int(I_1I_2)^{1/2}cos(\varphi_1 - \varphi_2)dxdy$. The first-order correlation term is the *positive* or *negative extra energy*. At the phase condition $\varphi_1 - \varphi_2 = \pi$, the interference completely *destroys* ($W = 0$) the energy. The system *creates* energy ($W = 4W_0$) in the case of $\varphi_1 - \varphi_2 = 0$. The same phase conditions provide the creating or destroying of energy by quantum two-source interference (for example, see formulas 4.A.1-4.A.9 [24]). The phenomenon depends neither on the nature (light or matter) of the waves (continuous waves or pulses) nor on material and shape of the subwavelength apertures (1-D and 2-D apertures or fibres). There is an evident resemblance between our model and a Dicke superradiance quantum model [25] of emission of an ensemble of atoms. A quantum reformulation of our model, which will be presented in our next paper, help us to understand better why a quantum entangled state is preserved on passage through a hole array [26].

It should be stressed that energy in conventional physical systems can be converted from one form to another, but it cannot be created or destroyed. According to our model, energy may be created or destroyed by constructive or destructive interference of waves (beams) only at the extremely particular phase conditions. The interference completely destroys energy if waves interfere destructively in all points of a physical system. The interference creates energy if waves interfere only constructively. The experimental realization of such phase conditions is practically impossible in conventional physical systems. We showed that the waves generated by the point-like sources separated by the distance $\Lambda < \lambda$ (for example, subwavelength gratings) can satisfy the phase conditions in the far-field diffraction zone.

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